

## NOTATION

$\lambda$ ,  $d$  are the channel length and diameter;  $d_s$ , diameter at the source exit;  $m$ , molecular mass;  $v_{\parallel}$ ,  $v_{\perp}$ , longitudinal and transverse components of the molecule thermal velocity;  $T_{\parallel}$ ,  $T_{\perp}$ , longitudinal and transverse temperatures;  $u$ , mass flow rate;  $Kn$ ,  $Kn_0$ , Knudsen numbers;  $T_d$ , temperature in the tank space;  $P_0$ ,  $T_0$ , pressure and temperature in the source chamber;  $A_1$ ,  $A_2$ ,  $B_1$ ,  $B_2$ , coefficients and exponents in relationships (2) and (3);  $\Sigma(x, \infty)$ , number of mutual molecule collisions;  $k$ , Boltzmann constant;  $\gamma$ , adiabatic index;  $n_0$ , density in the source chamber.

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## PARTICLE TEMPERATURE FLUCTUATIONS IN A TURBULENT STREAM

F. N. Lisin

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The question of temperature fluctuations of particles suspended in a turbulent stream of a radiating and scattering medium is analyzed.

Investigation of the fluctuation characteristics of phases [1] is important in the study of the heat transfer of two-phase turbulent streams. The purpose of the present report is investigation of the influence of radiation on turbulent particle temperature fluctuations.

The following assumptions were made: turbulence is homogeneous and isotropic, the particle sizes are sufficiently small and their heat conductivity is sufficiently large so that the temperature field within would remain homogeneous.

We write the particle energy equation in the form

$$\rho_p V_p c_p \frac{dT_p}{dt} = \frac{Nu \alpha_f}{d} s(T_f - T_p) + \frac{s}{4} \int_0^{\infty} k_a^{\lambda}(r) (G_{\lambda} - 4\pi I_{b\lambda}) d\lambda. \quad (1)$$

The spectral coefficient of particle absorption is computed according to the theory of Mie [2].

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Let us consider first the case of a gray medium with constant particle absorption coefficient over the spectrum. For dimensionless temperatures (1) takes the form

$$\frac{d\Theta_p}{dt} = A(\Theta_f - \Theta_p) + B(\Theta_f^4 F - \Theta_p^4), \quad (2)$$

where

$$\Theta = \frac{T}{T_0}; \quad A = \frac{3}{2} \frac{\text{Nu}\kappa_f}{c_p \rho_p r^2}; \quad B = \frac{3k_a \sigma_0 T_0^3}{c_p \rho_p r}.$$

The quantity  $F$  is found from solving the radiant energy transport equation. We use the solution in a  $P_1$  approximation by the method of spherical harmonics to perform the calculations. For a layer with transparent boundaries under isotropic scattering, we have from [3]

$$F = 1 - \frac{\text{ch} \left[ \sqrt{3(1-\gamma)} \left( \frac{\tau_0}{2} - \tau \right) \right]}{\text{ch} \left[ \sqrt{3(1-\gamma)} \frac{\tau_0}{2} \right] + \frac{2}{3} \sqrt{3(1-\gamma)} \text{sh} \left[ \sqrt{3(1-\gamma)} \frac{\tau_0}{2} \right]}. \quad (3)$$

We represent the temperature for a turbulent stream in the form of the sum of the average and the fluctuation components  $T = \langle T \rangle + T'$ .

Substituting this latter relationship into (2), taking the average and subtracting the equation obtained from the preceding one, we obtain an equation for the fluctuations. Neglecting terms therein that contain fluctuations in powers higher than the first, and setting approximately  $\langle T_p \rangle \approx \langle T_f \rangle$ , we find

$$\frac{d\Theta'_p}{dt} = A(\Theta'_f - \Theta'_p) + B_1(F\Theta'_f - \Theta'_p), \quad (4)$$

where

$$B_1 = 4B \langle \Theta^3 \rangle.$$

We represent the fluctuations in (4) in the form of stochastic Fourier-Stieltjes integrals

$$\Theta'_f = \int_{-\infty}^{\infty} \exp(i\omega t) dZ_{\Theta_f}(\omega); \quad \Theta'_p = \int_{-\infty}^{\infty} \exp(i\omega t) dZ_{\Theta_p}(\omega) \quad (5)$$

with random integrating functions. Substituting (5) into (4), we obtain for the differentials of the latter

$$dZ_{\Theta_p} = \frac{A + B_1 F}{i\omega + (A + B_1)} dZ_{\Theta_f}. \quad (6)$$

In conformity with the customary method [4], we find by going over to the spectral densities and integrating over the frequency  $\omega$

$$\langle \Theta_p'^2 \rangle = \langle \Theta_f'^2 \rangle \int_0^{\infty} \frac{(A + B_1 F)^2}{\omega^2 + (A + B_1)^2} f_{\Theta}(\omega) d\omega, \quad (7)$$

where  $f_{\Theta}(\omega)$  is the spectral function of the temperature fluctuations of the continuous phase.

We take an expression from [1] for the isotropic turbulence case as  $f_{\Theta}(\omega)$

$$f_{\Theta}(\omega) = \frac{2\pi\beta}{\sigma \sqrt{\langle u^2 \rangle}} \exp \left[ -\frac{(2\pi\omega)^2 \beta^2}{4 \langle u^2 \rangle \sigma^2} \right], \quad (8)$$

where  $\beta$  is the Lagrange microscale of turbulence,  $\sigma^2 = 1/3$ .

Substituting (8) into (7) and evaluating, we obtain

$$\frac{\langle \Theta_p'^2 \rangle}{\langle \Theta_f'^2 \rangle} = \sqrt{\pi} \left( \frac{A + B_1 F}{A + B_1} \right)^2 \psi \exp(\psi^2) \text{erfc } \psi, \quad (9)$$

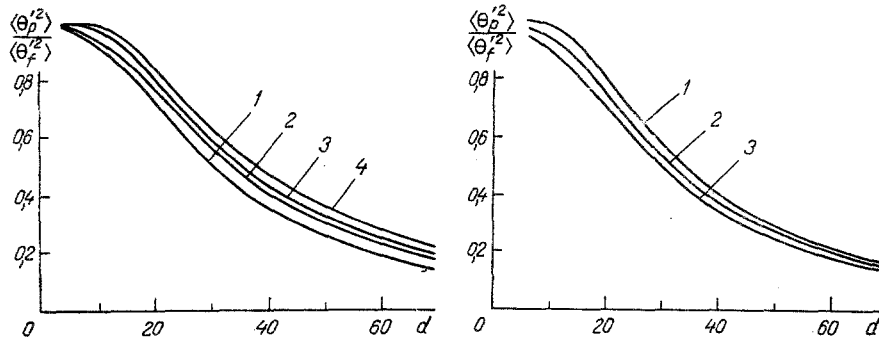


Fig. 1

Fig. 2

Fig. 1. Dependence of the ratio of the particle and fluid temperature fluctuations on the particle size: 1)  $\tau_0(1-\gamma) = 0.2$ ; 2) 0.5; 3) 1.0; 4)  $\infty$ .  $d$ ,  $\mu\text{m}$ .

Fig. 2. Influence of shifting of the absorption band edge by the quantity  $\langle \theta_p'^2 \rangle / \langle \theta_f'^2 \rangle$ : 1)  $\lambda_g = 0.5 \mu\text{m}$ ; 2) 1.0; 3) 2.0.

where

$$\psi = \frac{\pi\beta}{V \langle u^2 \rangle \sigma} (A + B_1).$$

Computations using (9) are performed for  $c_p = 0.837 \text{ kJ}/(\text{kg}\cdot\text{deg})$ ;  $\rho_p = 3000 \text{ kg}/\text{m}^3$ ;  $\kappa_f = 0.1 \text{ W}/(\text{m}\cdot\text{deg})$ ;  $\text{Nu} = 2$ ;  $B_1 = 10.6$ ;  $\pi\beta/(\sqrt{\langle u^2 \rangle \sigma}) = 2.6 \cdot 10^{-7}$ ;  $\beta \sim 10^{-3}$ ;  $u' = 0.2 \bar{u}$ ;  $\bar{u} = 50 \text{ m}/\text{sec}$ .

Since the  $P_1$ -approximation of the method of spherical harmonics is used which yields good accuracy for large  $\tau_0$  and  $\gamma$ , then the computations were performed for  $\tau_0 \geq 2$  and  $\gamma > 0.8$ . For an arbitrary scattering index the transition is realized by means of the substitution [5, 6]

$$\gamma_{\text{sph}} = \gamma \frac{(1 - \bar{\mu})}{(1 + \gamma\bar{\mu})}; \quad \tau_{\text{sph}} = \tau(1 - \gamma\bar{\mu}).$$

Shown in Fig. 1 is the influence of the particle size on the ratio between the rms fluctuations in the middle of the layer  $\tau = \tau_0/2$ .

As the optical thickness grows, the particle temperature fluctuations are subjected more strongly to temperature fluctuations of the carrying medium. And this is explained by the circumstance that for small  $\tau_0$  the particle interacts with the radiation field of the whole layer while as  $\tau_0$  grows this interaction becomes more local. The particle takes part in radiant heat transfer only with close-lying macrovolumes of the medium.

The selectivity was investigated approximately for two classes of substances: semiconductors and dielectrics. As is known, characteristic for them is the presence of a long-wavelength intrinsic absorption band edge in the visible or near-infrared spectrum domain [7]. The absorption edge was modeled as follows: for  $\lambda < \lambda_g$ ,  $\gamma_\lambda = \gamma_1$ ;  $k_a^\lambda = k_1$ ; for  $\lambda > \lambda_g$ ,  $\gamma_\lambda = \gamma_2$ ;  $k_a^\lambda = k_2$ . For such a modeling the radiant term in (1) has the form

$$\pi S \left\{ \int_0^{\lambda_g} k_1 [F_1 I_{b\lambda}(T_f) - I_{b\lambda}(T_p)] d\lambda + \int_{\lambda_g}^{\infty} k_2 [F_2 I_{b\lambda}(T_f) - I_{b\lambda}(T_p)] d\lambda \right\}, \quad (10)$$

$$F_1 = F(\tau_0; \gamma_1); \quad F_2 = F(\tau_0; \gamma_2).$$

We represent the radiation intensity in the form

$$I_{b\lambda}(T) \approx I_{b\lambda}(\langle T \rangle) + \frac{\partial I_{b\lambda}(\langle T \rangle)}{\partial T} T'. \quad (11)$$

Substituting (11) into (10) and performing the necessary calculations, we obtain (as before we assume approximately  $\langle T_f \rangle \approx \langle T_p \rangle$ ):

$$\frac{\langle \Theta_p'^2 \rangle}{\langle \Theta_f'^2 \rangle} = V \sqrt{\pi} \left( \frac{A + b_1 F_1 + b_2 F_2}{A + b_1 + b_2} \right)^2 \psi \exp(\psi^2) \operatorname{erfc} \psi,$$

where

$$b_1 = B_1 k_1 f_{0-\lambda}^*; \quad b_2 = B_1 k_2 (1 - f_{0-\lambda}^*);$$

$$\psi = \frac{\pi \beta}{V \langle u^2 \rangle \sigma} (A + b_1 + b_2),$$

$f_{0-\lambda}^*$  is a radiation function of the second kind [3].

Assumed in the computations was the attenuation coefficient  $k_0 = 2$ , ( $\rho = 2\pi r/\lambda > 10$ ), scattering coefficient  $k_p = \begin{cases} 1.6; & \lambda < \lambda_g, \\ 1.96; & \lambda > \lambda_g, \end{cases}$   $T_0 = 3000$  K. The remaining parameters are exactly the same as in the case of the gray medium.

Shown in Fig. 2 are dependences of  $\langle \Theta_p'^2 \rangle / \langle \Theta_f'^2 \rangle$  on the particle size for different  $\lambda_g$ . The shift of the absorption edge into the infrared spectrum domain for  $\tau_0 = \text{const}$  results in a smaller subjection of the particle temperature fluctuations to the continuous phase fluctuations. Seemingly suppression of the particle temperature fluctuations occurs because of the growing influence of radiation. For large particle sizes the absorption edge shift is insignificant. The magnitude of the particle temperature fluctuations enters into the terms  $\langle v_p' T_p' \rangle$  ( $v_p'$  is the particle velocity fluctuation) which is the turbulent energy transport by particles, when describing heat transfer in a turbulent flow of gas suspensions. The results obtained above indicate that as radiation grows, suppression of turbulent heat transport occurs to some degree, which is in agreement with the theoretical results of Ievlev [8].

#### NOTATION

$\rho_p, V_p, c_p$ , particle density, volume, and specific heat;  $d = 2r$ , particle diameter;  $\kappa_f$ , continuous phase heat conductivity;  $k_a \lambda$ , particle spectral absorption coefficient;  $G_\lambda$ , volume radiation energy density;  $I_{b\lambda}$ , Planck function;  $\lambda$ , radiation wavelength;  $Nu = \alpha d / \kappa_f$ ;  $\alpha$ , heat elimination coefficient;  $\theta = T/T_0$ ;

$$A = \frac{3}{2} \frac{Nu \kappa_f}{c_p \rho_p r^2}; \quad B = \frac{3 k_a \sigma_0 T_0^3}{c_p \rho_p r};$$

$\gamma$ , albedo of unitary scattering;  $\tau_0$ , optical thickness;  $\omega$ , frequency;  $\langle u'^2 \rangle$ , rms velocity fluctuation of the carrying medium;  $\bar{\mu}$ , mean cosine of the scattering index;  $f_{0-\lambda}^*$ , a radiation function of the second kind;  $v_p'$ , fluctuating component of the particle velocity. Subscripts: p and f, particle and fluid, respectively.

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